

71. Задачи од 3-1

1. Помножи ги мономите:

а) $-5a^2b^2$ и $-2a^3b$

$$(-5a^2b^2) \cdot (-2a^3b) =$$

$$= (-5)(-2)a^2a^3b^2b = 10a^5b^3$$

б) xy^3 и $-3a^2x^3y^2$

$$(xy^3) \cdot (-3a^2x^3y^2) =$$

$$= -3a^2xx^3y^3y^2 = -3a^2x^4y^5$$

в) $0,4a^{n-1}b^2$ и $5a^2b$

$$(0,4a^{n-1}b^2)(5a^2b) =$$

$$= 2,0a^{n-1}a^2b^2b =$$

$$= 2a^{n+1}b^3$$

г) $3a^{2m}b^{n+1}$ и $-8a^{m+1}b^n$

$$(3a^{2m}b^{n+1})(-8a^{m+1}b^n) =$$

$$= -24a^{2m}a^{m+1}b^{n+1}b^n =$$

$$= -24a^{2m+m+1}b^{n+1+n} =$$

$$= -24a^{3m+1}b^{2n+1}$$

2. Изврши го множењето:

а) $(8x^3 - 4x^2y^2 - 5xy^2 + 3y^3) \cdot (-2x^2y)$

$$(8x^3 - 4x^2y^2 - 5xy^2 + 3y^3) \cdot (-2x^2y) =$$

$$= -16x^5y + 8x^4y^3 + 10x^3y^3 - 6x^2y^4$$

б) $(4ab^2c - 7a^2bc^2 - a^2bc) \cdot (-3abc)$

$$(4ab^2c - 7a^2bc^2 - a^2bc) \cdot (-3abc) =$$

$$= -12a^2b^3c^2 + 21a^3b^2c^3 + 3a^3b^2c^2$$

3. Помножи ги полиномите:

а) $(x^2 - xy + 2y + 3x)$ и $(x - 4y + 5)$

$$(x^2 - xy + 2y + 3x) \cdot (x - 4y + 5) =$$

$$= \underbrace{x \cdot x^2 - x \cdot xy + x \cdot 2y + x \cdot 3x}_{4 \text{ членови}} - \underbrace{4y \cdot x^2 + 4y \cdot xy - 4y \cdot 2y - 4y \cdot 3x}_{4 \text{ членови}} + \underbrace{5 \cdot x^2 - 5 \cdot xy + 5 \cdot 2y + 5 \cdot 3x}_{4 \text{ членови}} =$$

$$= x^2 - x^2y + 2xy + 3x^2 - 4x^2y + 4xy^2 - 8y^2 - 12xy + 5x^2 - 5xy + 10y + 15x =$$

$$= -5x^2y + 9x^2 + 4xy^2 - 15xy + 15x - 8y^2 + 10y$$

б) $(3a^4 - 6a^3b + 5a^2b^2 - 7ab^3 - 9b^4)$ и $(a^2 - 3ab + b^2)$

$$(3a^4 - 6a^3b + 5a^2b^2 - 7ab^3 - 9b^4)(a^2 - 3ab + b^2) =$$

$$3a^6 - 6a^5b + 5a^4b^2 - 7a^3b^3 - 9a^2b^4 - 9a^5b + 18a^4b^2 -$$

$$-15a^3b^3 + 21a^2b^4 + 27ab^5 + 3a^4b^2 - 6a^3b^3 + 5a^2b^4 - 7ab^5 - 9b^6 =$$

$$= 3a^6 - 15a^5b + 26a^4b^2 - 28a^3b^3 + 17a^2b^4 + 20ab^5 - 9b^6$$

в) $\left(\frac{x}{2} + \frac{y}{3} + \frac{z}{4}\right)$ и $\left(\frac{x}{3} - \frac{y}{2} - \frac{z}{6}\right)$.

$$\left(\frac{x}{2} + \frac{y}{3} + \frac{z}{4}\right)\left(\frac{x}{3} - \frac{y}{2} - \frac{z}{6}\right) = \frac{x^2}{6} + \frac{xy}{9} + \frac{xz}{12} - \frac{xy}{4} - \frac{y^2}{6} - \frac{yz}{6} - \frac{xz}{12} - \frac{yz}{18} - \frac{z^2}{24} =$$

$$= \frac{x^2}{6} - \frac{y^2}{6} - \frac{z^2}{24} + \frac{xy}{9} - \frac{xy}{4} - \frac{yz}{8} - \frac{yz}{18} + \frac{xz}{12} - \frac{xz}{12} = \frac{x^2}{6} - \frac{y^2}{6} - \frac{z^2}{24} - \frac{5xy}{36} - \frac{13yz}{72}$$

или

$$\left(\frac{x}{2} + \frac{y}{3} + \frac{z}{4}\right)\left(\frac{x}{3} - \frac{y}{2} - \frac{z}{6}\right) = \frac{1}{12}(6x + 4y + 3z) \cdot \frac{1}{12}(4x - 6y - 2z) =$$

$$= \frac{1}{144}(24x^2 - 36xy - 12xz + 16xy - 24y^2 - 8yz + 12xz - 18yz - 6z^2) =$$

$$= \frac{1}{144}(24x^2 - 24y^2 - 6z^2 - 36xy + 16xy - 8yz - 18yz - 12xz + 12xz) =$$

$$= \frac{1}{144}(24x^2 - 24y^2 - 6z^2 - 20xy - 26yz) = \frac{1}{72}(12x^2 - 12y^2 - 3z^2 - 10xy - 13yz)$$

4. Докажи ги следните идентитети:

a) $(A+B+C)^2 = A^2 + B^2 + C^2 + 2AB + 2AC + 2BC$

$$L = (A+B+C)(A+B+C) =$$

$$A^2 + AB + AC + AB + B^2 + BC + AC + BC + C^2 =$$

$$A^2 + B^2 + C^2 + 2AB + 2AC + 2BC = D$$

б) $(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$

$$L = (A+B)(A+B)(A+B) =$$

$$= (A^2 + 2AB + B^2)(A+B) =$$

$$= A^3 + A^2B + 2A^2B + 2AB^2 + AB^2 + B^3 =$$

$$= A^3 + 3A^2B + 3AB^2 + B^3 = D$$

в) $(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$

$$L = (A-B)(A-B)(A-B) = (A^2 - 2AB + B^2)(A-B) =$$

$$= A^3 - A^2B - 2A^2B + 2AB^2 + AB^2 - B^3 = A^3 - 3A^2B + 3AB^2 - B^3 = D$$

5. Најди ги следниве квадрати и кубови:

a) $(x-5)^2$

$$(x-5)^2 = x^2 - 2 \cdot x \cdot 5 + 5^2 =$$

$$= x^2 - 10x + 25$$

б) $(3c+2)^2$

$$(3c+2)^2 = (3c)^2 + 2 \cdot 3c \cdot 2 + 2^2 =$$

$$= 9c^2 + 12c + 4$$

в) $(1-3x)^2$

$$(1-3x)^2 = 1^2 - 2 \cdot 1 \cdot 3x + (3x)^2 =$$

$$= 1 - 6x + 9x^2$$

г) $(3x-y+5)^2$

или види долу

$$(3x-y+5)(3x-y+5) =$$

$$= 9x^2 - 3xy + 15x - 3xy + y^2 - 5y + 15x - 5y + 25 =$$

$$= 9x^2 + 30x - 6xy - 10y + y^2 + 25$$

г) $(3x-y+5)^2$

$$[(3x-y)+5]^2 = (3x-y)^2 + 2(3x-y) \cdot 5 + 25 =$$

$$= 9x^2 - 2 \cdot 3x \cdot y + y^2 + 30x - 10y + 25 =$$

$$= 9x^2 - 6xy + y^2 + 30x - 10y + 25$$

д) $(x+y-z)^2$

$$[(x+y)-z][(x+y)-z] = (x+y)^2 - 2(x+y)z + z^2 =$$

$$= x^2 - 2xy + y^2 - 2xz + 2yz + z^2$$

ѓ) $(-8y^2-7z)^2$

$$(-8y^2-7z)^2 = (-1)^2(8y^2+7z)^2 =$$

$$= 1 \cdot (64y^4 + 2 \cdot 8y^2 \cdot 7z + 49z^2) =$$

$$= 64y^4 + 112y^2z + 49z^2$$

е) $(2a+b)^3$

$$(2a+b)^3 =$$

$$= (2a)^3 + 3 \cdot (2a)^2 \cdot b + 3 \cdot 2a \cdot b^2 + b^3 =$$

$$= 8a^3 + 12a^2b + 6ab^2 + b^3$$

ж) $(x-3y)^3$

$$(x-3y)^3 =$$

$$= x^3 - 3x^2 \cdot 3y + 3x \cdot 9y^2 - 27y^3$$

$$= x^3 - 9x^2y + 27xy^2 - 27y^3$$

з) $(2-0,5c)^3$

$$(2-0,5c)^3 = 8 - 3 \cdot 4 \cdot 0,5c + 3 \cdot 2 \cdot 0,25c^2 - 0,125c^3 = 8 - 6c + 1,5c^2 - 0,125c^3$$

6. Пресметај на најбрз начин:

$$42^2 = (40+2)^2 =$$

a) $= 40^2 + 2 \cdot 40 \cdot 2 + 2^2 =$

$$= 1600 + 160 + 4 = 1764$$

$$29^2 = (30-1)^2 =$$

б) $= 30^2 - 2 \cdot 30 \cdot 1 + 1^2 =$

$$= 900 - 60 + 1 = 841$$

$$75^2 = (100-25)^2 =$$

в) $= 10000 - 5000 + 625 =$

$$= 5625$$

г) $9,5^2 = (10-0,5)^2 =$

$$= 100 - 10 + 0,25 = 9,25$$

д) $198^2 = (200-2)^2 =$

$$= 40000 - 800 + 4 = 39204$$

7. Изврши го множењето на најбрз начин:

$$\begin{aligned} \text{a) } (5x+3y)(5x-3y) &= \\ (5x+3y)(5x-3y) &= \\ &= (5x)^2 - (3y)^2 = \\ &= 25x^2 - 9y^2 \end{aligned}$$

$$\begin{aligned} \text{б) } (1-a)(1+a) &= \\ (1-a)(1+a) &= \\ &= 1^2 - a^2 = \\ &= 1 - a^2 \end{aligned}$$

$$\begin{aligned} \text{в) } (2c-0,5)(2c+0,5) &= \\ (2c-0,5)(2c+0,5) &= \\ &= (2c)^2 - (0,5)^2 \\ &= 4c^2 - 0,25 \end{aligned}$$

$$\text{г) } 62 \cdot 58$$

$$\begin{aligned} 62 \cdot 58 &= (60+2)(60-2) = \\ &= 3600 - 4 = 3596 \end{aligned}$$

$$\text{д) } 7,01 \cdot 6,99$$

$$\begin{aligned} 7,01 \cdot 6,99 &= \\ &= (7+0,1)(7-0,1) = \\ &= 49 - 0,01 = 48,99 \end{aligned}$$

$$\text{ѓ) } 87^2 - 13^2$$

$$\begin{aligned} 87^2 - 13^2 &= \\ &= (87+13)(87-13) = \\ &= 100 \cdot 74 = 7400 \end{aligned}$$

8. Докажи ги следниве идентитети:

$$\begin{aligned} \text{a) } (a-b)^2 &= (b-a)^2 \\ L &= (a-b)^2 = a^2 - 2ab + b^2 = \\ &= b^2 - 2ba + a^2 = (b-a)^2 = D \end{aligned}$$

$$\begin{aligned} \text{б) } (-a-b)^2 &= (a+b)^2 \\ L &= (-a-b)^2 = [(-1)(a+b)]^2 = \\ &= (-1)^2 (a+b)^2 = (a+b)^2 = D \end{aligned}$$

$$\text{в) } (x-1)(x+1)(x^2+1) = x^4 - 1$$

$$L = (x-1)(x+1)(x^2+1) = (x^2-1)(x^2+1) = x^4 - 1 = D$$

$$\text{г) } x^3 + 3xy(x+y) + y^3 = (x+y)^3$$

$$L = x^3 + 3xy(x+y) + y^3 = x^3 + 3x^2y + 3xy^2 + y^3 = (x+y)^3 = D$$

9. Изврши ги назначените операции и упрости ги изразите:

$$\text{a) } x(x+2) \cdot (x-2) - (x-3)(x^2+3x+9)$$

$$\begin{aligned} x(x+2) \cdot (x-2) - (x-3)(x^2+3x+9) &= x(x^2-4) - (x^3 + \cancel{3x^2} + 9x - \cancel{3x^2} - 9x - 27) = \\ &= x^3 - 4x - x^3 + 27 = -4x + 27 \end{aligned}$$

$$\text{б) } (a+b+c) \cdot (a+b-c) - (a+b)^2$$

$$\begin{aligned} (a+b+c) \cdot (a+b-c) - (a+b)^2 &= [(a+b)+c] \cdot [(a+b)-c] - (a+b)^2 = \\ &= (a+b)^2 - c^2 - (a+b)^2 = -c^2 \end{aligned}$$